PROJECT 1

Power law channel geometry

ABSTRACT

In this project, students needed to develop an excel spreadsheet for the calculations of specific energy, specific momentum, critical depth, normal depth, and conjugate depth through the VBA codes.

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Background

In this project, a general spreadsheet with multiple worksheets to calculate different kinds of parameters for power-law section channel has been created. The worksheet works for both SI and engineering units. The worksheet could be used for calculating normal depth, critical depth at different flow rates, and conjugate depth for a hydraulic jump in horizontal channel. It could also be used for calculating and plotting the specific energy (E) diagram, as well as calculating and plotting the specific momentum diagram.

First of all, the power-law section is described as:

$$y = k\,x^m$$

In order to complete above tasks, the basic parameters such as top width (T), flow area (A), wetted perimeter (P) and Y_cA should be determined as a function of y. In this project, power-law section channel would be dealing with so numerical integration should be used for calculating geometrical parameters instead of traditional method.

According to the documents provided by Dr. Fang, the top width at a water depth (y) is given as:

$$T = 2\,f^{-1}(y)$$

Where this equation could be written for power-law section as:

$$x = (y/k)^{1/m}$$

Therefore, the top width T at any depth could be calculated as:

$$T = 2\,x = 2\,(y/k)^{1/m}$$

The coding work

To find the flow area (A) at any water depth (y), the integrate method would be applied as the equation shown below:

$$A = 2\int_{o}^{T/2} dA_i = 2\int_{o}^{T/2} \frac{y_i + y_{i+1}}{2}\,dx = dx[(y_0 + y_{100}) + 2\sum_{i=1}^{99} y_i]$$

In order to calculate the area shown above, considering Excel Solver need to be applied in the future, public function of VBA code should be used.

To set up the VBA tool in the Excel, go to “file”, “options”, “customize Ribbon” and adding check on “developer”. Go back to worksheet, choose “DEVELOPER” shown in toolbar.

To start editing the public function in VBA, click the “visual basic” and add “module” in this page. The public function should be editing under “module”.

For calculating the area, the name of public function should be named like “Area_cal” to avoid the same name with the other functions already exists. Then the title of this function should be:

“Public function Area_cal (k, dx, m)”

Where “k”, “dx”, “m” are the parameters need to be selected in worksheet which already been calculated by using basic equations.

Trapezoidal rule was used to do the numerical integration to get the flow area. 100 interval was made so dx=(T/2)/100. As a function of x_i, there’s one hundred and one yi need to be calculated (y_0 is also included).

In the public function, array was used because it’s easier to manage the yi value. A loop was used to calculate the value from yi(1) to yi(101). (In this project, yi(1) represents y_0, yi(2) represents y_1, the same way for the others.)

After calculating the yi value, they should be plug into another loop to calculate the area as shown below:

```
For i = 2 To 100
        sum = sum + Yi(i)
Next i
A_cal = dx * (Yi(1) + Yi(101) + 2 * sum)
```

Using the same idea, the wetted perimeter (P) and Y_cA could be calculated by using the public function. The reference equations are shown below:

$$P = 2\sum_{i=1}^{100} dp = 2\sum_{i=1}^{100}\sqrt{dx^2+dy^2} = 2\sum_{i=1}^{100}\sqrt{dx^2+(y_i-y_{i-1})^2}$$

$$Y_c A = 2\sum_{i=1}^{100} y_{ci}A_i = 2\sum_{i=1}^{100}\frac{y-y_i}{2}\frac{y_{i-1}+y_i}{2}dx \quad or \quad 2\sum_{i=1}^{100}\frac{y-y_i}{2}y_{i-1}dx$$

Results

Calculating and plotting the specific energy diagram

To calculate the specific energy (E), the equation shown below was used:

$$E = y + \frac{V^2}{2g}$$

Where v could be calculated by using v=Q/A.

Then various specific energy were calculated by following the changing y value. And the plot could be generated as the figure with corresponded results were shown below:

y (water depth)	X	T	dx	A (area)	V	E
(ft)	(ft)	(ft)	(ft)	(ft²)	(ft/s)	(ft)
0.20	0.2	0.40	0.00	0.04	125.00	242.82
0.37	0.3725	0.75	0.00	0.14	36.03	20.54
0.55	0.545	1.09	0.01	0.30	16.83	4.95
0.72	0.7175	1.44	0.01	0.51	9.71	2.18
0.89	0.89	1.78	0.01	0.79	6.31	1.51
1.06	1.0625	2.13	0.01	1.13	4.43	1.37
1.24	1.235	2.47	0.01	1.53	3.28	1.40
1.41	1.4075	2.82	0.01	1.98	2.52	1.51
1.58	1.58	3.16	0.02	2.50	2.00	1.64
1.75	1.7525	3.51	0.02	3.07	1.63	1.79

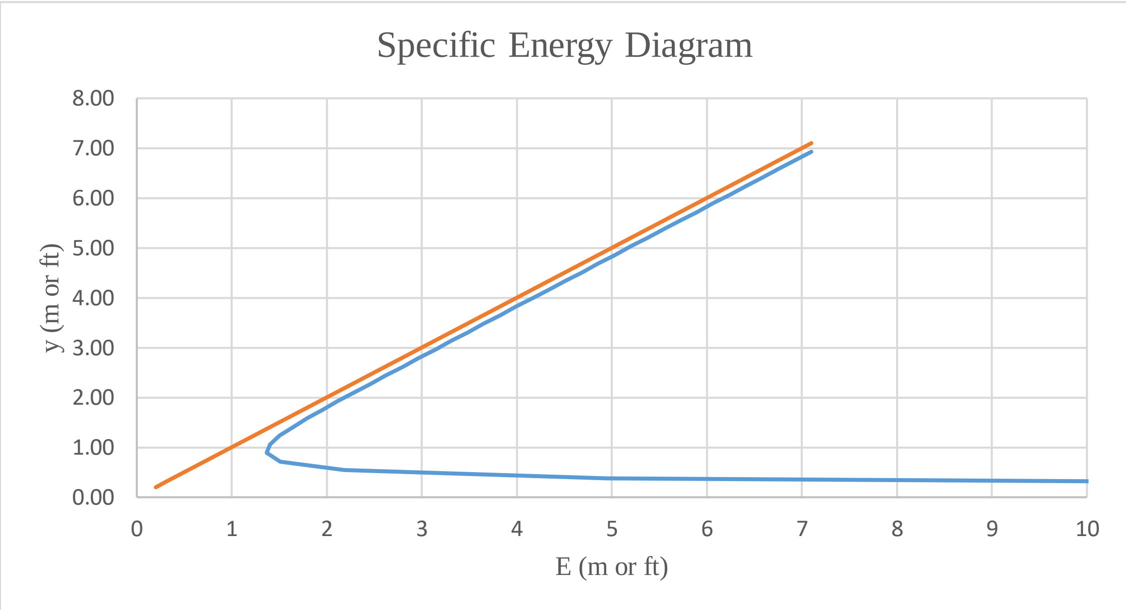

Figure 1. The Specific Energy results and diagram got from VBA code

In this case, m was set as 1, k was set as 1, Q was set as 5. Which as aim to verify with the standard spreadsheet provided by Dr. Fang. Because in this case, once the m was set as 1, the power-law channel would become a triangle channel. It could help to verify the calculation easily. The figure shown below displayed results calculated from Dr. Fang's spreadsheet:

13	y (water depth)	A (area)	V	E	T	Fr
14	(ft)	(ft²)	(ft/s)	(ft)	(ft)	
15	0.20	0.04	125.00	242.82	0.40	69.66
16	0.38	0.14	35.09	19.49	0.76	14.23
17	0.56	0.31	16.23	4.65	1.11	5.43
18	0.73	0.54	9.32	2.08	1.47	2.71
19	0.91	0.83	6.04	1.48	1.82	1.58
20	1.09	1.18	4.23	1.37	2.18	1.01
21	1.27	1.60	3.12	1.42	2.53	0.69
22	1.44	2.08	2.40	1.53	2.89	0.50
23	1.62	2.62	1.91	1.68	3.24	0.37
24	1.80	3.23	1.55	1.83	3.60	0.29
25	1.98	3.90	1.28	2.00	3.95	0.23
26	2.15	4.63	1.08	2.17	4.31	0.18
27	2.33	5.43	0.92	2.34	4.66	0.15
28	2.51	6.29	0.80	2.52	5.02	0.13
29	2.69	7.21	0.69	2.69	5.37	0.11
30	2.86	8.19	0.61	2.87	5.73	0.09
31	3.04	9.24	0.54	3.04	6.08	0.08
32	3.22	10.35	0.48	3.22	6.44	0.07
33	3.40	11.53	0.43	3.40	6.79	0.06

0.2	Minimum water depth
7.1	Maximum water depth
0.178	Increment of water depth (computed)

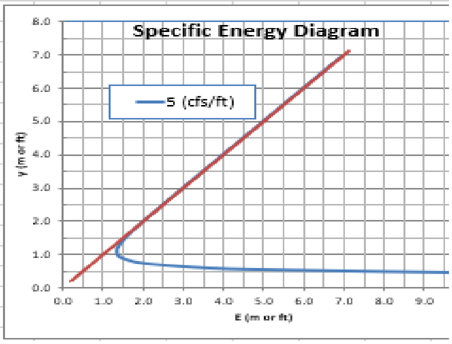

Figure 2. Results from Dr. Fang's spreadsheet

According to the result. The calculation were verified. The numbers were identical which means the calculations were reasonable.

Calculating and plotting the specific momentum diagram

To calculate the specific momentum diagram, the equation shown below was used:

$$M = \left(\frac{Q^2}{gA} + Y_C A \right)$$

Then various specific momentum were calculated by following the changing y value. And the plot could be generated as the figure with corresponded results were shown below:

y (water depth) (ft)	x (ft)	T (ft)	dx (ft)	A (area ft²	AY_C (ft³)	M (ft³)
0.50	0.50	1.00	0.01	0.25	0.04	310.60
0.73	0.73	1.45	0.01	0.53	0.12	147.83
0.95	0.95	1.90	0.01	0.90	0.28	86.30
1.18	1.18	2.35	0.01	1.38	0.52	56.76
1.40	1.40	2.80	0.01	1.96	0.89	40.50
1.63	1.63	3.25	0.02	2.64	1.39	30.79
1.85	1.85	3.70	0.02	3.42	2.05	24.73
2.08	2.08	4.15	0.02	4.31	2.89	20.92
2.30	2.30	4.60	0.02	5.29	3.93	18.61
2.53	2.53	5.05	0.03	6.38	5.21	17.38
2.75	2.75	5.50	0.03	7.56	6.73	16.99
2.98	2.98	5.95	0.03	8.85	8.52	17.29
3.20	3.20	6.40	0.03	10.24	10.60	18.18
3.43	3.43	6.85	0.03	11.73	12.99	19.61
3.65	3.65	7.30	0.04	13.32	15.73	21.55
3.88	3.88	7.75	0.04	15.02	18.82	23.99

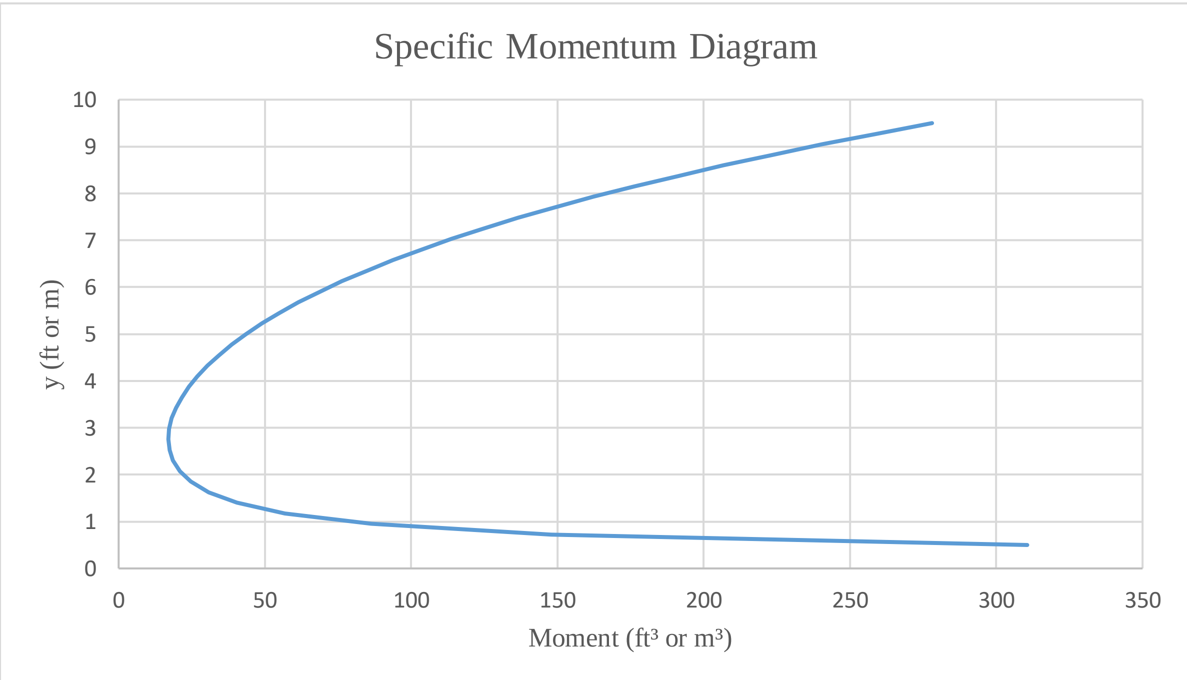

Figure 3. The specific momentum diagram created from VBA code

y (water depth) (ft)	A (area) (ft²)	AY_C (ft³)	M (ft³)
0.50	0.25	0.04	310.60
0.75	0.56	0.14	138.17
1.00	1.00	0.33	77.97
1.25	1.56	0.65	50.34
1.50	2.25	1.13	35.63
1.75	3.06	1.79	27.14
2.00	4.00	2.67	22.08
2.25	5.06	3.80	19.13
2.50	6.25	5.21	17.63

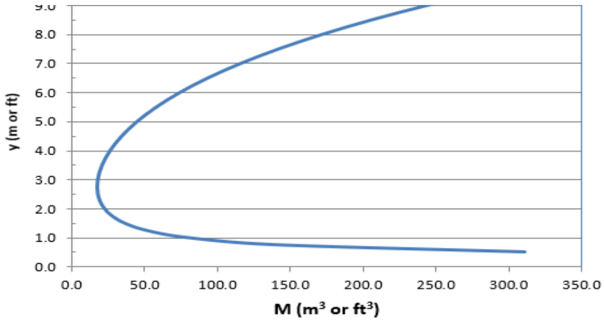

Figure 4. The results got from Dr. Fang's spreadsheet

According to the result. The calculation were verified. The numbers were identical which means the calculations were reasonable.

The normal depth

In this case, the Excel solver was used to minimize the difference between calculated flow (Q_1) and given flow (Q_2). The calculated flow (Q_1) was calculated by using the equation shown below:

$$Q = \frac{k_n}{n} \times A \times R^{2/3} \times {S_0}^{1/2}$$

The calculated flow (Q_1) was generated by using the guessing normal depth (y_n) with several known parameters. (k_n, n, A, R, S_0) Then Excel solver was applied to calculated difference between calculated flow and given flow as 0 solving for normal depth (y_n).

The example used to verify is shown below:

Example 1. A power-law channel is characterized with the following parameters: $H = 4$ m, $k = 0.75$ m^{1-m}, $S_0 = 0.001$, $n = 0.025$ $s/m^{1/3}$, and $Q = 10$ m^3/s. Then, compute the normal depth for $m = 1.1$, 1.25, 1.5, and 2.

To calculate the normal depth, the values of z, ε, and δ were computed using Eqs. (2), (14), and (16), respectively. Table 2 summarizes the results and shows the efficient performance of the proposed explicit solution.

m (—)	z [Eq. (2)] (—)	ε [Eq. (14)] (—)	δ [Eq. (16)] (—)	$\eta = (z^2\delta)^{m/(2m-2)}$ (—)	$y = \eta H$ (m)	y/k (m^m)
1.10	1.041	0.081	0.849	0.632	2.529	3.372

Figure 5. The example problem for normal depth

The results provided by Excel solver is shown below:

Normal depth	3.041295023	(m)			
			Units:	1	("1" for SI units or "0" for FPS)
m	1.1		Cosntant:	9.81	m^2/s
k	0.75		Kn	1	
Q	10	(m^3/s)	n	0.025	
X	3.570465221	(m)	So	0.001	
T	7.140930443	(m)			
dx	0.035704652	(m)			
A	10.34185556	(m^2)			
P	15.47341156	(m)			
YcA	10.48392391	(m^3)			
R	0.668362987	(m)			
V	0.966944465	(m/s)			
Calculated Q	10.00000123	(m^3/s)			
Comparison	1.23088E-06				
Difference compared to example:					
	10.87%				

Figure 6. The result calculated from the current project

According to the result. The calculation were verified. The numbers were identical which means the calculations were reasonable.

Critical depth

In this case, the Excel solver was used to set Froude number as 1 by changing critical depth. The Froude number (F_r) could be calculated as:

$$F_r = \frac{V}{\sqrt{gD}}$$

Where D=A/T.

The example used to verify is shown below:

	A	B	C	D	E	F	G
1	**Critical Depth Computation in a Trapezoidal Channel**						
2	***Note: Set Z_1 and Z_2 as zero for a Rectangular Channel***						
3	***Set B = 0 for a Triangular Channel***				***(Yellow cells for data entry)***		
4							
5	*System of Units: 1 for SI and 0 for FPS*			**1**	**SI units**		
6	*Acceleration of gravity (g)*			9.81	**m/s^2**		
7							
8	**Given Channel and Flow Parameters:**						
9	Channel Bottom Width (B)			0	**m**		
10	Left Side Slope (H:V, Z_1)			1			
11	Right Side Slope (H:V, Z_2)			1			
12	Discharge (Q)			20	**m^3/s**		
13							
14	**Critical Depth (y_c)**			2.411	**Adjustable Cell**		
15					**(Enter your estimate)**		
16	**Channel Geometrical Parameters:**						
17	Flow Area (A) = (2B+Z_1*y+Z_2*y)*y/2			5.82	**m^2**		
18	Top Width (T) = B+Z_1*y+Z_2*y			4.82	**m**		
19	Hydraulic Depth (D)= A/T			1.21	**m**		
20	Average Velocity (V_{avg}) = Q/A			3.44	**m/s**		
21	Froude Number Fr = V_{avg}/(gD)^0.5			1.00	**Target Cell**		
22	Depth ratio (y_c/B)			#DIV/0!			
23	Critical depth y_c from euqation(Swemee, 1993)			2.41			
24	Percent of error from the equation			-0.04	%		

Figure 7. The results calculated by Dr. Fang's spreadsheet

The results provided by current project is shown below:

Critical depth	2.411508694	(m)			
			Units:	1	("1" for SI units or "0" for FPS)
m	1		Costant:	9.81	m²/s
k	1				
Q	20	(m³/s)			
X	2.411508694	(m)			
T	4.823017387	(m)			
dx	0.024115087	(m)			
A	5.815374777	(m²)			
P	11.64403439	(m)			
YcA	4.53530407	(m³)			
R	0.499429543	(m)			
V	3.439159257	(m/s)			
D	1.205754471	(m)			
Fr	0.999973159				

$$F_r = \frac{V}{\sqrt{gD}}$$

$$D = \frac{A}{T}$$

Figure 8. The results calculated by current project

According to the result. The calculation were verified. The numbers were identical which means the calculations were reasonable.

Hydraulic Jump

Based on the momentum equilibrium equations shown below, with given parameters (Y_{J1}, m, k, Q), the specific momentum at different water depth were calculated. Excel solver was used to make $M_{J1}=M_{J2}$.

$$M_{j1} = M_2$$

$$\left(\frac{Q^2}{gA_{J1}} + Y_{CJ1}A_{J1}\right) = \left(\frac{Q^2}{gA_{J2}} + Y_{CJ2}A_{J2}\right)$$

The results provided by Dr. Fang's spreadsheet is shown below:

	A	B	C	D	E	F	G	H	I
1	Hydraulic Jump in a Trapezoidal Channel				**Units and Fluid Properties**				
2	***Note: Set Z_1 and Z_2 as zero for a Rectangular Channel***				***System of Units:***				
3	***Set B = 0 for a Triangular Channel***				1	(1 for SI and 0 for FPS)			
4				**Warning message:**	9.81	m/s²	Acceleration of gravity		
5	**Upstream depth y_1**	**1.00**	m		Specific weight				
6	**Downstream depth y_2**	**9.11**	m		9810	N/m³			
7	Channel bottom width (b)	0	m		Density	***(change fluid properties with ten***			
8	Left side slope Z_1 (H:V)	1			1000	kg/m³			
9	Right side slope Z_2 (H:V)	1							
10	Discharge	50	m³/s						
11	$(Qm^{1.5})/(g^{0.5}b^{2.5})$ - **Fig. 2.28**		$(m = Z_1 = Z_2)$						
12									
13	**Upstream**			**Downstream**					
14	Area A_1	1	m²	Area A_2	83.01	m²			
15	Depth at the centroid	0.33	m	Depth at the centroid	3.04	m			
16	Pressure at the centriod P_1	3270.00	pa	Pressure at the centriod P	29793.09	pa			
17	Velocity	50.00	m/s	Velocity	0.60	m/s			
18	$P_1A_1+\rho QV_1 = \gamma M_1$	2503270.00	N	$P_2A_2+\rho QV_2 = \gamma M_2$	2503270.00	N			
19	Froude Number F_{r1}	**22.58**		Froude Number F_{r2}	**0.09**				
20	Specific momentum (M_1)	255.175	m³	Specific momentum (M_2)	255.175	m³			

Figure 10. The results provided by Dr. Fang's spreadsheet

The results provided by current spreadsheet is shown below:

m	1	
k	1	
Q	50	(m^3/s)
X_1	1	(m)
T_1	2	(m)
dx_1	0.01	(m)
A_1	1	(m^2)
P_1	4.828529	(m)
YcA_1	0.3234	(m^3)
R_1	0.207102	(m)
V_1	50	(m/s)
D_1	0.5	(m)
Fr_1	22.57618	
M_{j1}	255.1654	(m^3)
y_{j2}	9.204009	(m)
X_2	9.204009	(m)
T_2	18.40802	(m)
dx_2	0.09204	(m)
A_2	84.71377	(m^2)
P_2	44.44179	(m)
YcA_2	252.157	(m^3)
R_2	1.906174	(m)
V_2	0.590223	(m/s)
D_2	4.602004	(m)
Fr_2	0.087843	
M_{j2}	255.1653	(m^3)
ΔM	7.91E-05	(m^3)

Units:	1	("1" for SI units or "0" for FPS)
Cosntant:	9.81	m^2/s
Kn	1	

$$\left(\frac{Q^2}{gA_{J1}} + Y_{CJ1}A_{J1}\right) = \left(\frac{Q^2}{gA_{J2}} + Y_{CJ2}A_{J2}\right)$$

$$M_{j1} = M_{j2}$$

Figure 11. The results provided by current project

According to the result. The calculation were verified. The numbers were identical which means the calculations were reasonable.

Conclusion

This spreadsheet combined by public function and solver can be used to calculate normal depth, critical depth, specific energy and specific momentum for any channel with power-law section when $1 \leq m \leq 2$. It can also be used to plot the specific momentum and specific energy diagrams. Furthermore, this spreadsheet could solve hydraulic jump problems for the power law channel, which can be practically used.

There were some assumptions considered in this project during the calculations. In this project, the discharge was always assumed as constant at different locations (as water depth changed). In the reality, the discharge was not controlled, which means this excel spreadsheet could not applied when the discharge was not constant. In addition, when momentum equation was applied during the calculation, friction force and gravity force were ignored since $S_0 \sim 0$. In the reality, this excel spreadsheet could not be used if S_0 is dramatically larger than 0, which needs to consider friction force and gravity force at that time.

Reference

http://eng.auburn.edu/users/xzf0001/CIVL5110/index.html

Ali R. Vatankhah, 2014. Normal depth in power-law channels. *ASCE Journal of Irrigational and Drainage Engineering.*